Noise Estimation in GRAPPA Reconstructed Data

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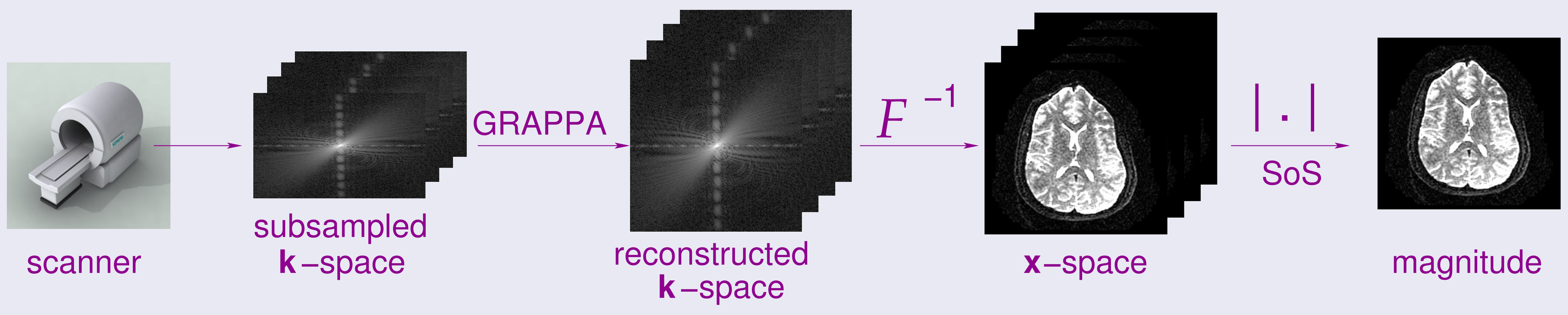
A new method to estimate the variance of noise from the composite magnitude signal of GRAPPA reconstructed images is presented. Parallel imaging methods allow to increase the acquisition rate via subsampled acquisitions of the k-space. However, the reconstruction process yields to a variance of noise value which is dependent on the position within the image. The proposed method uses information of the GRAPPA reconstruction coefficients and assumes a final non-central chi distribution to recover the spatial pattern of noise.

Noise statistical model in GRAPPA



The GeneRalized Autocalibrated Partially Parallel Acquisitions (**GRAPPA**) reconstruction strategy estimates the full **k**-space in each coil from a sub–sampled **k**-space acquisition. The reconstructed lines are estimated through a linear combination of the existing samples. Weighted data in a neighborhood $\eta(\mathbf{k})$ around the estimated pixel from several coils is used for such an estimation.

In [1] authors pointed out that the resultant distribution of the Composite Magnitude Signal is not strictly a nc-χ, but it could be modeled as such with a small approximation error. However, the final distribution will show a (reduced) *effective number of coils* L_{eff} and an (increased) *effective variance of noise* σ^2_{eff}:

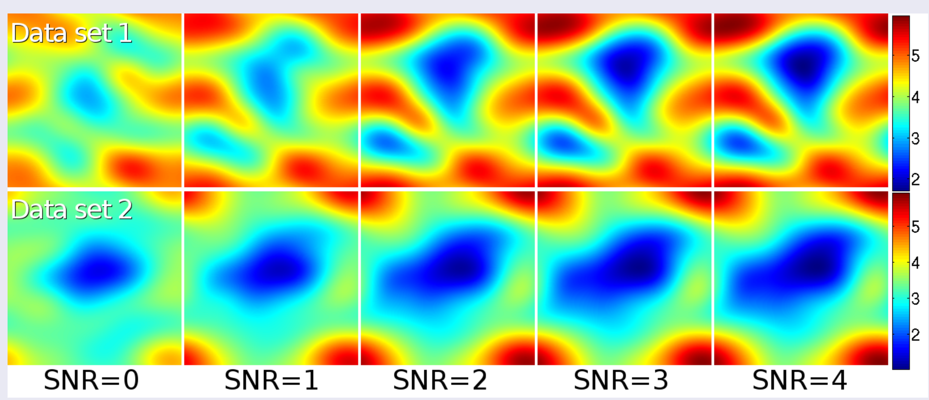
$$L_{\text{eff}}(\mathbf{x}) = \frac{|\mathbf{A}|^2 \,\mathrm{tr}\left(\mathbf{C}^2_X\right) + \left(\mathrm{tr}\left(\mathbf{C}^2_X\right)\right)^2}{\mathbf{A}^*\mathbf{C}^2_X\mathbf{A} + ||\mathbf{C}^2_X||^2_F}; \qquad (1)$$

$$\sigma^2_{\text{eff}}(\mathbf{x}) = \frac{\mathrm{tr}\left(\mathbf{C}^2_X\right)}{L_{\text{eff}}}. \qquad (2)$$

where $\mathbf{C}^2_X(\mathbf{x}) = \mathbf{W}\Sigma^2\mathbf{W}^*$ is the covariance matrix of the *interpolated* data at each spatial location, $\mathbf{A}(\mathbf{x}) = [A_1, \cdots, A_L]^T$ is the noise-free reconstructed signal, $||.||_F$ is the Frobenious norm, Σ^2 is the covariance matrix of the *original* data:

$$\Sigma^2 = \begin{pmatrix} \sigma_1^2 & \cdots & \sigma^2_{1L} \\ \vdots & \ddots & \vdots \\ \sigma^2_{L1} & \cdots & \sigma^2_L \end{pmatrix}$$

and $\mathbf{W}(\mathbf{x})$ is the GRAPPA interpolation matrix for each ($\mathbf{x}$). Although the nc-χ model is feasible for GRAPPA, the resulting distribution is non-stationary since the effective parameters are spatially dependent.



Simulated σ_n value for real GRAPPA coefficients for different SNR.

Noise estimation in GRAPPA

Background of the reconstructed image approximated by a c-χ, therefore

$$\mathrm{E}\{M_L^2\} = 2\sigma_n^2 L$$

Using Effective parameters:

$$\mathrm{E}\{M_L^2\}(\mathbf{x}) = 2\ \sigma^2_{\text{eff}}(\mathbf{x})\ L_{\text{eff}}(\mathbf{x}) = 2\ \mathrm{tr}\left(\mathbf{C}^2_X(\mathbf{x})\right)$$

Considering no correlation between coils and each coil corrupted with uncorrelated Gaussian noise with the same σ_n:

$$\mathbf{C}^2_X(\mathbf{x}) = \mathbf{W}(\mathbf{x})\cdot\Sigma^2\cdot\mathbf{W}^*(\mathbf{x}) = \sigma_n^2\cdot\mathbf{W}(\mathbf{x})\cdot\mathbf{W}^*(\mathbf{x})$$

We define $\mathscr{G}_W(\mathbf{x}) = \mathrm{tr}(\mathbf{W}(\mathbf{x})\mathbf{W}^*(\mathbf{x}))$, then

$$\mathrm{E}\{M_L^2\}(\mathbf{x}) = 2\ \sigma_n^2\ \mathscr{G}_W(\mathbf{x}) \qquad (3)$$

and

$$\sigma_n^2 = \frac{\mathrm{E}\{M_L^2\}(\mathbf{x})}{2\ \mathscr{G}_W(\mathbf{x})} \qquad (4)$$

Following the estimation philosophy in [2,3] we define a noise estimator based on sample second order moment $\langle M_L^2(\mathbf{x})\rangle_{\mathbf{x}}$ (Gamma distributed) and then

$$\mathrm{mode}\left\{\frac{\langle M_L^2\rangle_{\mathbf{x}}}{\mathscr{G}_W}\right\} = 2\sigma_n^2 \frac{\mathscr{G}_W}{\mathscr{G}_W L_{\text{eff}}}\frac{|\eta(\mathbf{x})|L_{\text{eff}} - 1}{|\eta(\mathbf{x})|}, \approx 2\sigma_n^2$$

The estimator is then defined as

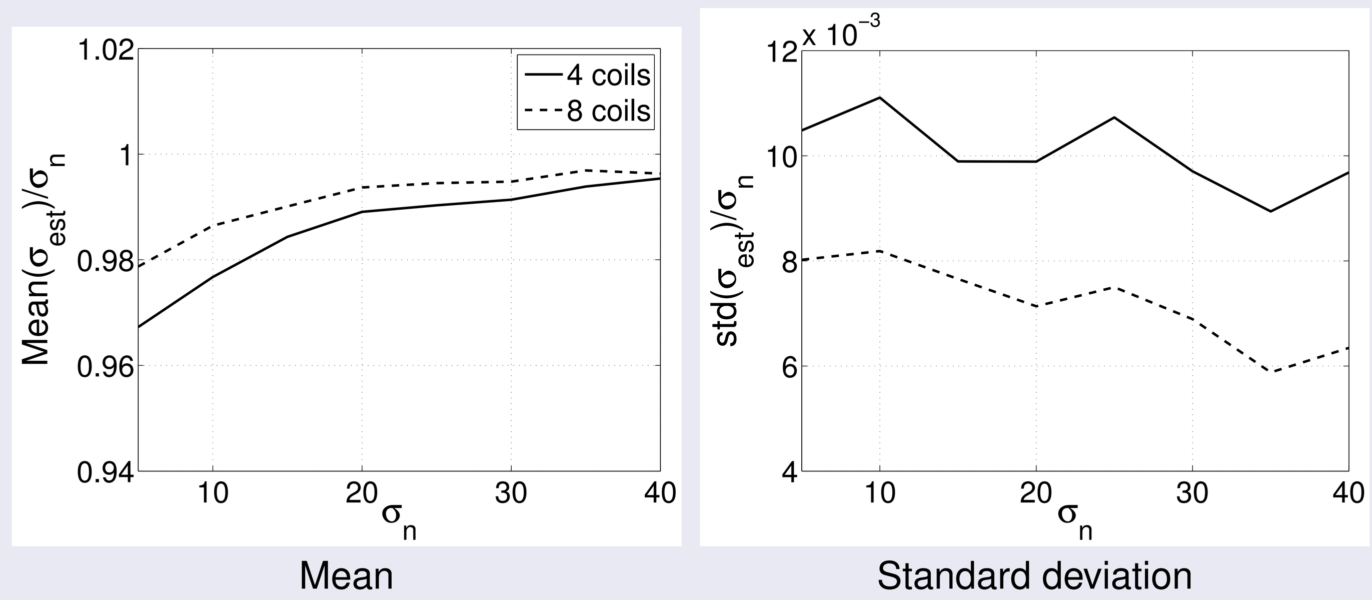
$$\widehat{\sigma_n^2} = \frac{1}{2}\mathrm{mode}\left\{\frac{\langle M_L^2(\mathbf{x})\rangle_{\mathbf{x}}}{\mathscr{G}_W(\mathbf{x})}\right\} \qquad (5)$$

In the background we can determine $\sigma^2_{\text{eff}}(\mathbf{x})$:

$$\sigma^2_{\text{eff}}(\mathbf{x}) = \widehat{\sigma_n^2}\frac{||\mathbf{W}(\mathbf{x})\mathbf{W}^*(\mathbf{x})||^2_F}{\mathscr{G}_W(\mathbf{x})} \qquad (6)$$

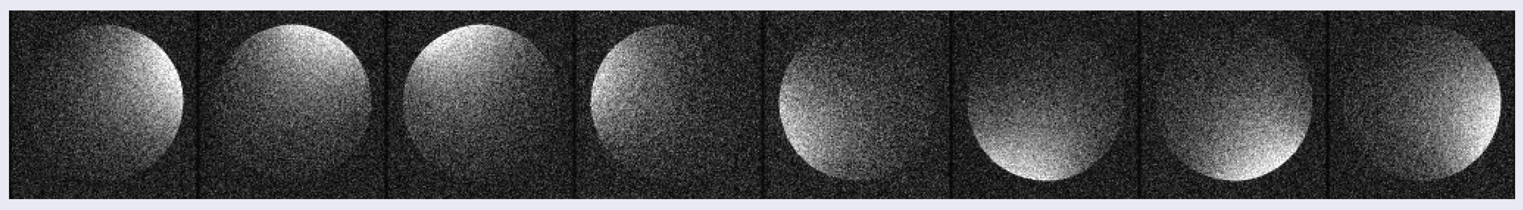
Experiments and Results

Synthetic Experiment: 2D slice from BrainWeb, intensity values in $[0-255]$ (Averages: White Matter 158, Gray Matter 105, cerebrospinal fluid 36, the background 0). 4- and 8-coil systems where simulated using an artificial sensitivity map. Each coil corrupted with Gaussian noise with $\sigma_n \in [5-40]$. The **k**-space is uniformly subsampled by a factor of 2, with 32 ACS lines.

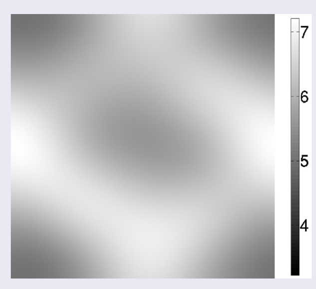


Results of σ_n estimation using the proposed method; 100 experiments are considered for each sigma value.

Real acquisitions: 100 repetitions of a phantom, scanned in 8-coil GE Signa 1.5T EXCITE 12m4 scanner with FGRE Pulse Sequence to generate low SNR.



$2\times$ subsampled and GRAPPA reconstructed. Coefficients are derived from one sample, using 32 ACS lines. $\mathscr{G}^2_W(\mathbf{x})$ values derived from the GRAPPA coefficients:



Noise variance σ_n^2 is estimated: (1) From the real part of every coil of every sample. This value σ_0^2 is taken as Golden Standard. (2) Over the final GRAPPA reconstructed images, using the estimator proposed in eq. (5).

σ_0	mean$\{\widehat{\sigma_n}\}$	mean$\{\widehat{\sigma_n}\}/\sigma_0$	std$\{\widehat{\sigma_n}\}/\sigma_0$
0.0428	0.0424	0.9905	0.0113

Results obtained estimating the noise with the proposed method is totally consistent with the value obtained over the complex Gaussian images without subsampling. There is a very small bias in the estimation and the method also shows a very small variance, as also seen in the synthetic experiments.

Conclusions

- If the complex data in each coil before reconstruction were available, the estimation could be done using a simple Gaussian noise estimator.
- Although the estimation of the original variance of noise σ_n^2 is of great importance *per se*, the spatially variant pattern of noise can also be inferred; or at least the product $\sigma^2_{\text{eff}}(\mathbf{x})L_{\text{eff}}(\mathbf{x})$, which is the parameter usually needed by algorithms.
- The estimation method has shown to be robust and easy to use.
- Limitations: (1) Correlation between coils has not been taken into account. (2) GRAPPA reconstruction coefficients must be known. (3) Processing software in the scanner may add a mask to data, which eliminates part of the background, drastically reducing the number of points available for noise estimation.

Acknowledgements

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